Fostering Success in Mathematics for English Learners with the Common Core State Standards (CCSS): Recommendations and Resources for Supporting Academic Literacy in Mathematics

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1. Introduction

The goals of this paper are to describe recommendations and resources for fostering success for English Learners in mathematics classrooms aligned with the Common Core. In the first section of the paper, I summarize research-based recommendations relevant to supporting success for ELs in mathematics classrooms aligned with the CCSS. I synthesize what research tells us about effective mathematics instruction, supporting classroom mathematical discourse, and addressing needs specific to English Learners (ELs) in mathematics classrooms. In the second section of the paper I describe three resources for framing research and instruction that focuses on academic literacy in mathematics: work on mathematical discourse, the ELP Framework (CCSSO, 2012; Lee et al, 2013), and a socio-cultural framework for academic literacy in mathematics (Moschkovich, in preparation). I describe two approaches to academic literacy in mathematics that are informed by work in mathematical discourse and recognize the complexity of classroom mathematical discourse. One approach is the ELP framework mathematics sections that used a complex view of mathematical discourse to describe the language tasks in mathematics classrooms. The other approach is a socio-cultural framework for academic literacy in mathematics (Moschkovich, in preparation), used here to analyze the mathematical proficiency, mathematical practices, and mathematical discourse involved in solving a mathematics word problem. These two frameworks can be used to design instruction and assessment, improve instruction, or review materials intended to support the success of ELs in mathematics classrooms.

2. Recommendations for fostering success of ELs in mathematics classrooms

In this section I summarize four sets of research-based recommendations that are relevant to fostering success for ELs in mathematics classrooms:

- Recommendations for mathematics instruction aligned with the CCSS
- Recommendations for supporting classroom mathematical discourse
- Recommendations for supporting academic success for ELs
- Recommendations for addressing the needs of ELs specific to mathematics instruction

I first provide a summary of what the CCSS mean for mathematics instruction, focusing on four emphases in the CCSS and the characteristics of effective instruction in mathematics (for all students, not ELs in particular). I then summarize research-based recommendations for supporting academic success for ELLs (that are not specific to mathematics). I compare the three sets of recommendations to describe intersections and provide a final set of recommendations that are specific to mathematics instruction for ELs.

The CCSS in mathematics call for a shift away from traditional practices for teaching mathematics. For example, the eight standards for mathematical practice described in the CCSS require students to be actively engaged in mathematical practices such as making sense of problems, modeling, and constructing and critiquing arguments. Instruction as envisioned in the CCSS is expected to support mathematical discussions and use a variety of participation structures (teacher led, small group, pairs, student presentations, etc.) that allow students to use multiple representations (diagrams, charts, symbols, models, etc.) in communicating about mathematical content as they engage in these mathematical practices. This emphasis on mathematical practices places new language demands on students to learn to participate in mathematical activity that includes language and other semiotic symbol systems to talk, read, write, problem solve, and think like mathematicians.

Aligning with the CCSS

What are the characteristics of mathematics instruction that is aligned with the CCSS? First and foremost, mathematics instruction that is aligned with the CCSS means teaching mathematics for understanding. In general, when the focus is on understanding, students actively use and connect multiple representations, they develop meaning for symbols, and have opportunities to share and refine their explanations, conjectures, reasoning, justifications, and arguments. These characteristics are based not only on the CCSS but also on research-based recommendations for effective mathematics instruction. According to a review of the research (Hiebert & Grouws, 2007), mathematics teaching that makes a difference in student achievement and promotes conceptual development in mathematics has two central features: 1) teachers and students attend explicitly to concepts, and 2) teachers give students the time to wrestle with important mathematics. Another research based recommendation is to use and maintain high cognitive demand mathematical tasks, for example, by encouraging students to explain their problem-solving and reasoning (AERA 2006; Stein, Grover, & Henningsen 1996).

Below are four general recommendations for mathematics instruction that is aligned with the CCSS:

RECOMMENDATIONS FOR MATHEMATICS INSTRUCTION ALIGNED WITH THE CCSS

1. **Balance conceptual understanding and procedural fluency.** Instruction should balance student activities that address important conceptual and procedural knowledge and connect the two types of knowledge.
2. **Maintain high cognitive demand.** Instruction should use high cognitive demand math tasks and maintain the rigor of tasks throughout lessons and units.
3. **Develop beliefs.** Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable.
4. **Depth before breadth.** Instruction should go in depth into fewer topics rather than less depth for more topics.
5. **Engage students in mathematical practices.** Instruction should provide opportunities for students to engage in these mathematical practices: Make sense of problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; model with mathematics; use appropriate tools strategically; attend to precision; look for and make use of structure; look for and express regularity in repeated reasoning.

Supporting classroom mathematical discourse

The shift in the CCSS towards mathematical practices requires attention to mathematical discourse. Many valued academic mathematical practices involve mathematical discourse. Students learn to communicate mathematically by making conjectures, presenting explanations, and constructing arguments that involve mathematical objects, with mathematical content, and towards a mathematical point (Brenner, 1994). Participating in mathematical discourse is essential for ELs to learn both content and language and therefore, recommendations for supporting classroom mathematical discourse are essential for supporting ELLs in mathematics classrooms.

Focusing on classroom mathematical discourse can shift how we frame the dilemma of teaching ELs mathematics. A focus on classroom mathematical discourse shifts the problem from “I teach math, not language” to the question “How can I teach math through language (and other symbols systems).” Work in mathematical discourse should inform instruction for ELs because this work brings crucial research on classrooms discussions, genres of mathematical texts, and, overall, a complex view of mathematical discourse as multimodal and multi-semiotic (O’Halloran, 2000, 2005). This work thus recognizes and addresses the complexity of language in math classrooms and focuses on teaching practices to support students in engaging in this complexity. Work on mathematical discourse provides a view of language in math classrooms as complex and including multiple representations (objects, pictures, words, symbols, tables, graphs, etc.), modes (oral, written, receptive, expressive), types of written texts (textbooks, word problems, student explanations, teacher explanations), kinds of talk (exploratory and expository), and audiences (presentations to teacher, peers, by teacher, by peers). Instruction that supports classroom mathematical discourse is designed to support students in engaging with this complexity.

Below are recommendations for how mathematics instruction can support classroom mathematical discourse:

RECOMMENDATIONS FOR SUPPORTING CLASSROOM MATHEMATICAL DISCOURSE

Provide students opportunities for multiple

1. Resources: Representational (gestures, objects, etc.), linguistic (everyday language), cognitive (invented algorithms),

2. Modes: speaking, listening, reading, writing, drawing, graphing, etc.

3. Purposes: Students describe, compare, explain, argue, articulate ideas, interpret information, share explanations, present solutions, defend claims, etc.

4. Participation structures: Students participate in work alone, in pairs, small groups, presentation, teacher-led discussions, etc.

Explanations and justifications need not be only expressed in words. Instruction should certainly support students in learning to develop oral and written explanations, but students can show conceptual understanding by using pictures (for example of a rectangle as an area model to show that two fractions are equivalent or how multiplication by a positive fraction smaller than one makes the result smaller). Students need opportunities to actively use and connect multiple representations, show and describe meaning for symbols, and share, refine, and critique their reasoning. Therefore, these recommendations require that teachers develop teaching strategies for providing opportunities for participating in mathematical discourse.

Skills for teaching mathematics through participation in classroom mathematical disocurse are fundamental to supporting students in the CCSS, the standards for mathematical practices, and teaching math for understanding[1]. For example, teachers need the skills and strategies for leading, supporting, and orchestrating mathematical discussions, whether these occur in small groups or with the whole class. These teaching strategies are best learned in the context of a particular mathematics topic-- for example learning what the best questions are to support algebraic thinking (Driscoll, 1999), or geometric thinking (Driscoll, 2007). These strategies are also best learned through long-term professional development that engages teachers in observation, watching video, sharing lessons, etc.

Supporting academic success for ELLs

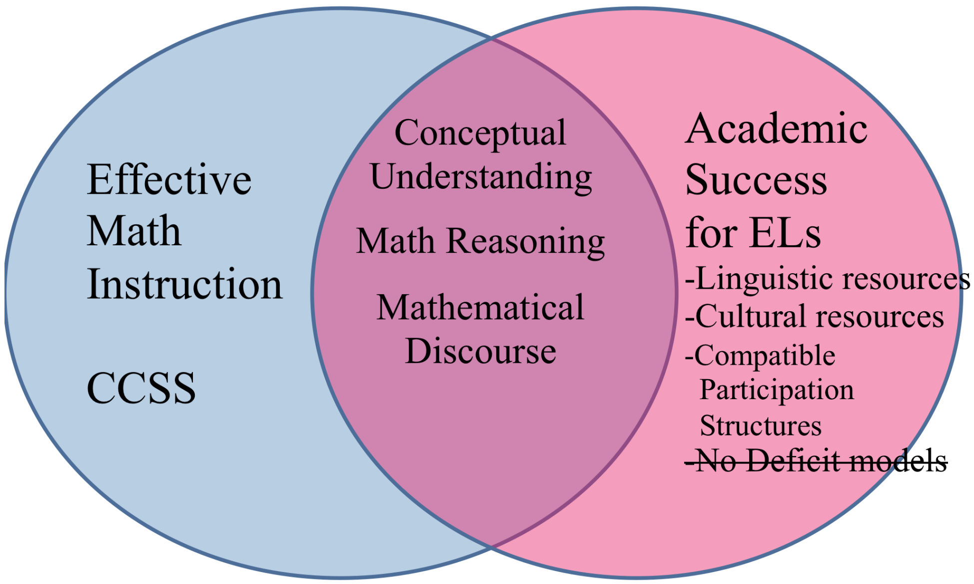
Although it is difficult to make generalizations about the instructional needs of all students who are learning English, instruction should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). In addition, research suggests that high-quality instruction for ELLs that supports student achievement has two general characteristics: a view of language as a resource rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009). Research provides general guidelines for instruction for this student population. Overall, students who are labeled as ELLs are from non-dominant communities and they need access to curricula, teachers and instructional techniques proven to be effective in supporting the academic success of these students. The general characteristics of such environments are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction should "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (Garcia & Gonzalez, 1995, p. 424).

RECOMMENDATIONS FOR SUPPORTING ACADEMIC SUCCESS FOR ELS

1) Instruction is based on rigorous, standards-based curriculum; provide additional time & instruction, but not lower expectations (AERA Research Points, 2004)
2) Instruction treats language as a resource, not a deficit, and emphasizes academic achievement, not only learning English (Gándara & Contreras, 2009)
3) Teachers reject deficit models of students, hold high expectations for all students, can change curriculum and instruction to meet specific needs of students (Garcia & Gonzalez, 1995)
4) Provide opportunities for multiple modes (speaking, listening, reading, and writing); encourage students to develop understanding, construct meaning, and refine interpretations (Garcia & Gonzalez, 1995

The recommendations for supporting academic success for ELs overlap in several important ways with the CCSS, with recommendations for effective mathematics teaching, and with recommendations for supporting classroom mathematical discourse. This means that neither policy nor practitioners need to start from scratch to design mathematics instruction that will be aligned with the CCSS and also address the needs of ELs. The three sets of recommendations agree that student understanding, meaning construction, and reinterpretation are central for effective instruction. The recommendations informed by work in mathematical discourse intersect with those for ELs to including interactions that include multiple modes, resources, purposes, and situations, overall suggesting that there needs to be more than talk and text, that mathematical discussions need to be carefully orchestrated, and that mathematical discussions should occur not only during teacher led whole class situations but also during other classroom arrangements. Beyond these intersecting recommendations, are there recommendations that may be specific to mathematics instruction for ELs?

These recommendations for supporting the academic success of ELs overlap with recommendations for effective mathematics instruction and for mathematics teaching aligned with the CCSS. The fact that the three sets of recommendations summarized above share some common elements means that supporting success for ELs is compatible with mathematics instruction aligned the CCSS. At the very least, all three sets of recommendations point to conceptual understanding, mathematical reasoning, and mathematical discourse. Beyond these common elements, there may be some issues that are specific to the needs of ELs in mathematics classrooms or become more salient when teaching this student population. Below is a diagram showing points of intersection and issues that may be specific to supporting academic success for ELs:



Addressing needs of ELs specific to mathematics instruction

Mathematics instruction that fosters success for ELs should have the characteristics of effective mathematics instruction, fit the recommendations for aligning with the CCSS, support classroom mathematical discourse, and follow the recommendations for academic success for ELs. In addition, there are general guidelines for teaching mathematics to ELs and issues that are specific to the needs of ELs in mathematics classrooms.

Research on language and mathematics education provides some general guidelines for instructional practices for teaching ELs mathematics (Moschkovich, 2010). Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. Mathematics instruction for ELs should address much more than vocabulary and support ELs' participation in mathematical discussions as they learn English. Instruction should draw on the multiple resources available in classrooms (objects, drawings, graphs, and gestures), as described in the recommendations for supporting classroom mathematical discourse. In addition, instruction should also draw on the linguistic and cultural resources that are specific to ELs, such as their home or first language and alternative algorithms for arithmetic operations, and experiences outside of school that are specific to ELs.

There are some issues that are specific to mathematics instruction for ELs. Teachers need to be aware of the specific linguistic and cultural resources that ELs bring to mathematics. For example, cognates from Spanish that often show up in mathematical terms in geometry can support Spanish speakers understanding of formal mathematical vocabulary (translation, rotation, etc.) in contrast to more colloquial terms such as flip (Shannon, p.c.). Notation and algorithms differ across communities and these can sometimes make students seem slower or less competent in arithmetic computation. In order to know whether particular resources such as vocabulary in a first language, alternative algorithms, or notation, are re actually a part of a student's repertoire, teachers need to know the details of the history of their students' academic instruction in mathematics. Participation structures needs to be as compatible as possible with home and community norms (Au, 1980; Brenner, 1998).

The final recommendations summarized below are based largely on research that has contradicted deficit models of ELs as mathematics learners (Moschkovich, 2002) and called for a move away from vocabulary as the focus of instruction for ELs (Moschkovich, 2010). Teachers can provide students with opportunities to develop both language and mathematical competencies through instruction aligned with the three recommendations below (these are described in more detail in Moschkovich, 2012):

RECOMMENDATIONS FOR MATHEMATICS INSTRUCTION FOR ELLS
1. Focus on students' mathematical reasoning, not accuracy in using language.
2. Focus on mathematical practices, not language as vocabulary.
3. Treat everyday and home languages as resources, not obstacles.

Focus on students' mathematical reasoning, not accuracy in using language: Instruction should focus on uncovering, hearing, and supporting students' mathematical reasoning, not on accuracy in using language (Moschkovich, 2010). Instruction should focus on recognizing students' emerging mathematical reasoning and focus on the mathematical meanings learners construct, not the mistakes they make or the obstacles they face. Instruction needs to first focus on assessing content knowledge as distinct from fluency of expression in English, so that teachers can then build on, extend, and refine student's mathematical reasoning. If we focus only on language accuracy, we miss the mathematical reasoning.

Focus on mathematical practices, not language as words, vocabulary, or definitions**:** Instruction should move away from simplified views of language and interpreting "language" as vocabulary, single words, or a list of definitions (Moschkovich, 2010). An over-emphasis on correct vocabulary limits how we see and hear student competencies. If we only focus on accurate vocabulary, we can miss how students are participating in mathematical practices. Instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations. Instruction should provide opportunities for students to actively engage in mathematical practices such as reasoning, constructing arguments, expressing structure and regularity, etc. When designing instruction, consider how students will participate in the eight standards mathematical practice across the various modes of communication (reading, writing, listening, speaking) that students will be used during instruction. It is not necessary to include every practice in every lesson, the goal is to provide students opportunities to actively participate in these mathematical practices when possible and appropriate.

Treat everyday and home languages as resources, not obstacles: Treating home or everyday language as obstacles limits the linguistic resources for communicating mathematical reasoning. Everyday language and academic language are interdependent and related-- not mutually exclusive. Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics (Moschkovich, 2002, 2007c). All students, including ELLs, bring linguistic resources to the mathematics classroom that can be employed to engage with activities designed to meet the CCSS. As students continue to expand their linguistic repertoires in English, students can use a wide variety of linguistic resources-- including home languages, everyday language, developing proficiency in English, and nonstandard varieties of English-- to engage deeply with the kinds of instruction called for in the CCSS (Bunch, Kibler, & Pimentel, 2012).

3. Resources for framing academic literacy in mathematics

Implementing these recommendations requires frameworks that synthesize, organize, and operationalize how teachers and students participate in academic literacy in mathematics. The CCSS, with its emphasis on mathematical practices, coupled with the urgency to address the needs of ELLs provide an opportunity to develop theoretically grounded, comprehensive, and coherent approaches to academic literacy in mathematics. These approaches should draw on research from multiple relevant fields and honor the complexity of classroom mathematical discourse. In order to tackle the complex issue of mathematics instruction for this student population, approaches need to draw on current research in both literacy studies and mathematics education and, in particular, research on mathematical discourse. In this section I describe three resources for framing research and instruction that focuses on academic literacy in mathematics: work on mathematical discourse, the ELP Framework (CCSSO, 2012; Lee et al, 2013), and a socio-cultural framework for academic literacy in mathematics (Moschkovich, in preparation).

Shifting to a complex view of classroom mathematical discourse is crucial for fostering the success of ELs in mathematics. Research and policy have repeatedly, clearly, and strongly called for mathematics instruction for this student population to maintain high standards (AERA, 2004) and high cognitive demand (AERA, 2006). In order to accomplish these goals, instruction for ELs needs to move beyond defining academic literacy in mathematics as low-level language or arithmetic skills. Instruction needs to begin with high cognitive demand tasks that require conceptual understanding, provide opportunities for students to participate in mathematical practices, and support classroom mathematical discourse. To support academic literacy in mathematics instruction needs to enact a view of mathematical discourse includes multiple modes, symbol systems, registers, and languages. The first step in making these shifts is to use an expanded view of classroom mathematical discourse. The foundation for this expanded view lies largely in previous work on mathematical discourse.

Work on mathematical discourse

Many commentaries on the role of *academic language* in mathematics teaching reduce the meaning of the term to single words or vocabulary (for an example, see Cavanagh, 2005). In contrast, work on mathematical discourse provides a more complex view of mathematical discourse that should inform work on mathematics instruction for ELs. The shift from academic language to mathematical discourse is particularly important for definitions of academic literacy in mathematics for ELs. Research and practice on mathematics instruction fro ELs should draw on work in mathematical discourse. Research and instruction for this student population needs to move away from oversimplified views of language as words, phrases, vocabulary, or lists of definitions and leave behind an overemphasis on correct vocabulary and formal language. Such views severely limit the linguistic resources teachers and students can use in the classroom to learn mathematics with conceptual understanding and preclude students from participating in valued mathematical practices.

Work on the language of disciplines (e.g., Pimm, 1987) provides a complex view of mathematical language as not only specialized vocabulary---new words and new meanings for familiar words---but also as extended discourse that includes syntax and organization (Crowhurst, 1994), the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007c). Definitions of academic literacy in mathematics need to move beyond interpretations of the mathematics register as merely a set of words and phrases particular to mathematics. The mathematics register includes styles of meaning, modes of argument, and mathematical practices and has several dimensions of complexity, for example how mathematics texts are organized or how classroom mathematical discourse positions students.

Work on mathematical discourse brings a complex and detailed view of discourse in mathematics classrooms and has addressed multiple topics. As a start, this work assumes that learning mathematics is a discursive activity (Lerman, 2001; O'Connor, 1998; Pimm, 1987; Sfard, 2001). Other work has examined a multitude of topics, many of them relevant to mathematics classrooms with ELs, including mathematical texts (Morgan, 1998; O'Halloran, 2005), polysemy (Pimm, 1987), mathematical discourse at home (Walkerdine, 1988), negotiated meanings (Zack & Graves, 2001), sociomathematical norms and argumentation (Yackel & Cobb, 1996), classroom discussions (Herbel-Eisenmann, 2007; Herbel-Eisenmann, & Wagner, 2010; Wagner, & Herbel-Eisenmann, 2008), connections between discourse and equity (Herbel-Eisenmann et al, 2012), and equity issues in student participation (Esmonde & Moschkovich, 2012). This work has also analyzed curriculum from a discourse perspective (Herbel-Eisenmann, 2007) and teacher moves during classroom discussions (i.e. re-voicing in O'Connor & Michaels, 1993). More recently, work on mathematical discourse has provided resources for teachers to learn how to orchestrate classroom discussions (Chapin, O'Connor, & Anderson, 2003; Herbel-Eisenmann, 2002; Herbel-Eisenmann & Cirillo, 2009). Researchers have also examined mathematical discourse in bilingual and multilingual classrooms (Adler, 2001; Brenner, 1998; Barwell, Barton & Setati, 2007; Licon-Khisty, 1995; Setati, 2005; Moschkovich, 2002).

Work in mathematical discourse provides several contributions that are relevant to research and practice in mathematics classrooms with ELs. Overall, this work provides a view of mathematical discourse not as vocabulary or technical words but as the communicative competence necessary and sufficient for competent participation in mathematical practices. Work on mathematical discourse has described the multimodal and multi-semiotic nature of mathematical activity, how meanings are situated and negotiated, and how multiple registers co-exist in mathematics classrooms. This work provides a complex view of mathematical discourse as multimodal and multi-semiotic (O'Halloran, 2005; Radford, 2003; Moschkovich, 2008) and a shift from seeing mathematical language as having static meanings to views of meanings as situated, dynamic, and negotiated (Moschkovich, 2008; O'Connor, 1998; Zack & Graves, 2001). Another important contribution is the descriptions of classroom mathematical discourse as combining everyday, school, and academic registers (Forman, 1996; Moschkovich, 1998, 2007c). A contribution that is especially relevant to word problems used for assessments is a shift from focusing on mathematical language or the mathematics register at the word level. The following word problem illustrates how the mathematics register is not about technical vocabulary:

> A boat in a river with a current of 3 mph can travel 16 miles downstream in the same amount of time it can go 10 miles upstream. Find the speed of the boat in still water. (Borrowed and adapted from materials by G. Cook & R. MacDonald, WIDA UW)

The complexity involved in making sense of this word problem is not at the level of technical mathematical vocabulary, but lies principally in the background knowledge (Martiniello, 2008; Martiniello & Wolf, 2012; Solano-Flores, 2010) for understanding and imagining the context. In this case, the reader needs to imagine and understand that there is a boat traveling up and down a river, that the speed was measured in still water (presumably a lake), and that the speed of the boat increases (by the speed of the current) when going downstream, and decreases (by the speed of the current) when going upstream. The language complexity lies not in understand mathematical terms, but having the background knowledge to imagine the situation, upstream downstream, in still water. Although understanding words such as upstream, downstream, and the phrase "in still water" would be helpful, much of the language complexity is not at the word level, but at the sentence & paragraph level, in the use of the passive voice without an agent and in the multiple subordinate clauses and nested constructions, (Cook & MacDonald, 2012).

English Language Proficiency Framework

The ELP framework mathematics sections (CCSSO, 2012) reflect the assumptions, insights, and shifts described in the preceding section for work in mathematical discourse and provides a resource for considering how to enact a complex view of mathematical discourse in classrooms. The sections on mathematics describe the receptive and productive language tasks for teachers and students using a complex view of classroom mathematical discourse that includes multiple modes, registers, and participation structures. For example, Table 8: Discipline-specific language in K-12 mathematics classrooms (page 33), describes the complexity of multiple registers present in classrooms. Registers include not only discipline specific language and terminology, but also disciplinary discourse conventions along with classroom and colloquial registers, as well informal and informal written communication. This table also includes multiple participation structures, such as whole group, small group, in pairs, as well as interactions with adults and peers.

Insert Table 4: Math Table 8 in ELP Framework

This framework was designed to connect the mathematical practices described in the CCSS to different ways to use language in the classroom. The central question that connects the mathematical practices to language tasks is "What oral, written, receptive or productive language tasks are involved for teacher and students to participate in this mathematical practice?" For Math Practice #1, some of the associated receptive language functions are: comprehend the meaning of a problem as presented in multiple representations, such as spoken language, written texts, diagrams, drawings, tables, graphs, and mathematical expressions or equations, comprehend others' talk about math problems, solutions, approaches, and reasoning; coordinate texts and multiple representations. For Math Practice #1, the general productive language function is to communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to problem solving. More specific productive language functions are: create, label, describe, and use in presenting solutions to a math problem multiple written representations of a problem; explain in words orally or in writing relationships between quantities and multiple representations of problem solutions; present information, description of solutions, explanations, and arguments to others, respond to questions or critiques from others; and ask questions about others' solutions, strategies, and procedures for solving problems

Insert Tables 1-2-3: Math Tables in ELP Framework, Section B, Tables 1-2-3

A socio-cultural framework for academic literacy in mathematics

Defining and framing academic literacy in mathematics is more than a theoretical exercise. With a clear definition and a detailed framework, we can make recommendations for instruction and support a complex view of academic literacy in mathematics. Researchers and practitioners can also use this framework to analyze student and teacher activity; choose, design, or enact tasks to support academic literacy in mathematics; recognize academic literacy in mathematics in student activity, or assess student competencies and progress. Here I summarize a socio-cultural framework and use a word problem to illustrate how the framework makes visible the complexity of academic literacy in mathematics. This framework provides a way to insure attention to the multiple intertwined components of academic literacy in mathematics.

A socio-cultural perspective on academic literacy in mathematics shifts from simplified views of academic language and mathematical competence to a complex description of mathematical activity that includes not only mathematical knowledge, but also mathematical practices and mathematical discourse. This socio-cultural perspective of academic literacy in mathematics begins by providing a complex view of mathematical proficiency as participation in discipline-based practices that involve conceptual understanding and mathematical discourse. This framework includes mathematical practices and mathematical discourse as central components of academic literacy in mathematics and focuses on the complexity of both mathematical practices and mathematical discourse.

The socio-cultural theoretical framework draws on situated perspectives of learning mathematics (Brown, Collins and Duguid, 1989; Greeno, 1998). From this perspective, learning mathematics is a discursive activity (Forman, 1996) that involves participating in a community of practice (Forman, 1996; Lave and Wenger, 1991; Nasir, 2002), developing classroom socio-mathematical norms (Cobb et al., 1993), and using material, linguistic, and social resources (Greeno, 1998). This perspective assumes that participants bring multiple perspectives to a situation, that representations and utterances have multiple meanings for participants, and that these multiple meanings are negotiated through interaction. The situated nature of academic literacy in mathematics means that in classrooms the meanings of academic mathematical language are embedded in mathematical practices and in the local socio-cultural setting. The hybrid nature of academic literacy in mathematics means that mathematical practices involve not only oral and written text, but also include multiple modes and symbol systems. Similarly, mathematical discourse involves not only school mathematical language but also home languages and the everyday register as resources for mathematical reasoning.

In this framework, academic literacy in mathematics has three components: mathematical proficiency, mathematical practices, and mathematical discourse (Moschkovich, in preparation):

Insert Table 5 here: FRAMEWORK FOR ACADEMIC LITERACY IN MATHEMATICS

The first step in defining academic literacy in mathematics is to include the complexity of *mathematical proficiency* (for a summary of mathematical proficiency, followed by descriptions of a socio-cultural view of mathematical practices and mathematical discourse see Moschkovich, in preparation). A current description of mathematical proficiency comes from the book "Adding it up: helping children learn mathematics" published by the National Research Council (Kilpatrick, Swafford, & Findell, 2001). The NRC volume defines the five intertwined strands of mathematical proficiency: 1) Conceptual understanding (comprehension of mathematical concepts, operations, and relations; 2) Procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; 3) Strategic competence (competence in formulating, representing, and solving mathematical problems, especially novel problems, not routine exercises); 4) Adaptive reasoning (logical thought, reflection, explanation, and justification); and 5) Productive disposition (a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

If students are participating in academic literacy in mathematics as defined here, then we can see or hear them actively using concepts, showing conceptual understanding, and participating in mathematical practices, many of which are discursive. Since mathematical discourse is multimodal and multi-semiotic (O'Halloran, 2000. 2005), instruction supporting academic literacy in mathematics should include multiple modes, sign systems, and types of inscriptions. Since different participant structures support mathematical discussions, instruction for academic literacy in mathematics should include multiple arrangements for student participation. The framework thus serves to keep our attention on the complexity of academic literacy in mathematics.

The question "What oral, written, receptive or productive language tasks are evident (or possible) for teacher and students?" taken from the ELP Framework (CCSSO, 2012), provides the detail of language tasks for each mathematical practice in the CCSS. Although the ELP framework can inform instruction that addresses all three aspects of academic literacy in mathematics, the mathematics tables focused on making connections between mathematical practices and language task (see tables for MP #1-#2-#3).

This framework can be used to frame research analyses (see Moschkovich in preparation), consider curriculum materials, or enact instruction. As an example, the framework can be used to describe the academic literacy in mathematics involved in reading and making sense of the word problem shown below (Lappan et al, 1998).

Insert Figure 1: Problem from Connected Mathematics Project

What academic literacy in mathematics is involved in solving the first part of this problem, generating the graph? First, students need to read and understand the text that describes the situation. This text is different than texts in other content areas. The purpose of the text is not to tell a story, make an argument, or persuade the reader. Instead, the text provides a situation to be modeled using mathematics. The structure is that there is some information given that describes a real world situation and sets the stage, and then there are questions posed for the reader. The framework is especially useful in seeing how reading this word problem involves different literacy than reading non-word problems. In particular, the "reading" cannot be separated from the mathematical practice of "making sense" and using mathematical proficiency.

Students also need to read, understand, and use the information provided in the table. This is not as simple as it may seem. There are two typical interpretations of the second column that often arise in classrooms. One interpretation is that number in the second column refers to *interval distance*, i.e. that after 0.5 hours the bikers had traveled 8 miles, and after 1 hour the bikers had traveled an additional interval of 15 miles. Thus, after 1 hour, the bikers were not 15 miles away from their starting point, but 8 +15, or 23 miles from their starting point. The other interpretation is the second column refers to cumulative distance, i.e. that after 0.5 hours the bikers had traveled 8 miles, and after 1 hour the bikers had traveled a *cumulative distance* of 15 miles. Thus, after 1 hour, the bikers were 15 miles away from their starting point, not 23 miles. Students typically need to sort out which of these interpretations fits the situation. And lastly, students need to connect two representations by using the data in the table to construct a graph.

This task certainly involves mathematical proficiency beyond procedural fluency since generating the graph involves modeling with mathematics and making sense of three symbol systems (text, table, and graph), two central mathematical practices. Connecting these three representations is a typical way for a task to involve conceptual understanding (Moschkovich et al, 1993). However, the goal for this task is not determined only by what is given in the problem, it also depends on the activity structure provided by the classroom norms that require that students discuss their responses in small groups, arrive at joint group solutions, and present a group solution to the whole class. Without this activity structure this task would not necessarily engage students in mathematical practices that support classroom mathematical discourse, such as constructing viable arguments and critiquing the reasoning of others. Thus, the goal for any task depends in crucial ways on the norms established in each classroom for how students engage in mathematical discourse. Students would have the opportunity to participate in a collective discussion that involves the teacher and other students only be if classroom norms and the typical routine set the goals for the group work to serve as preparation for an ensuing discussion that involved not only individual sense making and reasoning, but also collectively sharing, comparing, and critiquing solutions as well as opportunities for negotiating meanings, rather than looking for single interpretations. In general, participation structures matter for enacting mathematical discourse. There are different types of talk, exploratory and presentational (Barnes, 2008) and students' mathematical work can be more visible during exploration in small groups than presentations in front of the whole class (Moschkovich, 2002b).

4. Conclusions

The recommendations and resources described here are evidence that it is possible to design mathematics instruction for ELs that is aligned with the Common Core. Recommendations for fostering success for English Learners in mathematics classrooms overlap in important ways with recommendations in the CCSS. In fact, teachers who have been learning how to orchestrate mathematical discussion in their monolingual classrooms can use some of those skills to work with ELs. What matters most is a complex view of classroom mathematical discourse. Research in mathematics education that has focused on mathematical discourse provides such a complex view and should be an integral part of work on developing instruction to foster the success of

ELs in mathematics. Two resources that draw on this research for framing academic literacy in mathematics are the ELP framework and a socio-cultural framework for academic literacy in mathematics (Moschkovich, in preparation). These two frameworks provide resources that can be used to design instruction and assessment, improve instruction, or review materials intended for ELs in mathematics classrooms. Future work using these two frameworks includes developing strategies, materials, and principles for supporting ELs in learning to read and make sense of word problems.

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Acknowledgements

FIGURES AND TABLES

Insert Tables 1-2-3: Math Tables in ELP Framework (CCSSO, 2012), Section B, Tables 1-2-3 (attached as pdf)

Insert Table 4: Math Table 8 in ELP Framework (attached as pdf)

TABLE 5: FRAMEWORK FOR ACADEMIC LITERACY IN MATHEMATICS

Math Proficiency	Math Practices	Math Discourse
Which strands of math proficiency are necessary (evident, or possible)? Can the task or activity be modified to include more strands or address one strand in more depth?	Which math practices are necessary (evident or possible) to solve the problem?	Which mathematical discourse practices are evident (or possible)?
	What participation structures are necessary (evident, or possible) to engage students in participation in those math practices?	What typical math texts are involved (or possible)? What modes, purposes, representations are evident (or possible)?
Does the task require high cognitive demand? What is necessary to maintain high cognitive demand? Can the task be modified to require higher cognitive demand?	Are additional math practices possible?	What oral, written, receptive or productive language tasks are evident (or possible) for teacher and students? (Question from ELP Framework)
		Are there any discourse resources that are specific to these students or their community? Are participation structures compatible with students' home community norms?

Figure 1: Problem-From Atlantic City to Lewes

On the second day of their bicycle trip, the group left Atlantic City and rode five hours South to Cape May, New Jersey. This time, Sidney and Sarah rode in the van. From Cape May, they took a ferry across the Delaware Bay to Lewes, Delaware. Sarah recorded the following data about the distance traveled until they reached the ferry.

Time (hours)	Distance (miles)
0.0	0
0.5	8
1.0	15
1.5	19
2.0	25
2.5	27
3.0	34
3.5	40
4.0	40
4.5	40
5.0	45

1. Make a coordinate graph of the (time, distance) data given in the table. 2. Sidney wants to write a report describing the day 2 of the tour. Using information from the table and the graph, what would she write about the days travel? Be sure to consider the following questions:

A. How far did the group travel in the day? How much time did it take them?
B. During which interval(s) did the riders make the most progress? The least progress?
C. Did the riders go further in the first half or the second half of the days' ride?

2. By analyzing the table, how can you find the time intervals when the riders made the most progress? The least progress? How can you find these intervals by analyzing the graph?

From Connected Mathematics Project, Moving Straight Ahead unit.
From Lappan, G., Fey, J. T., Fitzgerald, W. M., Freil, S. N., and Phillips, E. D. (1998). *Connected Mathematics.* White Plains, NY: Dale Seymour Publications.

ENDNOTES

[1] Classroom mathematical discourse was a central focus of NCTM publications during 1980-2000, but less attention has been paid to mathematical discourse since NCLB. There are materials (books, videos, etc.) available that, although they do not target ELLs in particular, can be used to support teachers in learning to orchestrate mathematical discussions, for example NCTM book "Orchestrating mathematical discussions."