Supporting ELLs in Mathematics:

Mathematics Tasks with Annotations and Other Resources for Implementing the Common Core State Standards

Developed for the Understanding Language Initiative

PREFACE[1]

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The goal of these materials is to illustrate how mathematics tasks that are aligned with the Common Core State Standards (CCSS) for Mathematics can be used to support mathematics instruction and the learning of English Language Learners (ELLs), at three grade spans (elementary, middle, and high school). We used or adapted tasks from two publicly accessible curriculum projects, Inside Mathematics and Mathematics Assessment Project.

The resources provided here are based on the premise that ELLs develop mathematical proficiency as well as the linguistic resources to express that proficiency by actively participating in mathematical practices and rigorous mathematical reasoning that is well scaffolded by instruction. The eight Common Core Standards for Mathematical Practice focus on key aspects of mathematical expertise. These eight standards set expectations for students to be engaged in apprenticeships in mathematical activities that, over time, simultaneously build their procedural fluency, conceptual understanding, and participation in mathematical reasoning and sense making (Moschkovich, 2012). The additional support offered to ELLs through the UL resources is intended to scaffold their participation in these activities.

These materials have been reviewed by the Understanding Language Initiative's Mathematics Work Group and by a group of expert reviewers (see pages 23-25). All materials will be online at the Understanding Language website: http://ell.stanford.edu.

These materials and recommendations for teaching practice are based on research findings that often run counter to commonsense notions of *language*. There are multiple uses of the terms *language*, *academic language*, or *the language of mathematics*. Many interpretations of these terms for teaching practice reduce the meaning of academic language in mathematics to single words and the proper use of grammar. In contrast, these materials use a more complex view of mathematical language as not only specialized vocabulary but also as extended discourse that includes syntax,

[1] This preface is an adapted version of the introduction to the Understanding Language ELA Unit.

organization, the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007).

From the perspective of the Understanding Language initiative, language is an activity. In these resources, "the language of mathematics" does not mean a list of vocabulary or technical words with precise meanings but the communicative competence necessary and sufficient for competent participation in mathematical discourse practices (Moschkovich, 2012).

Although learning vocabulary may be necessary, it is not sufficient. Learning to communicate mathematically and participate in mathematical discussions is not simply a matter of learning vocabulary. During discussions in mathematics classrooms, students are learning to describe relationships, make generalizations, and use representations to support their claims. The question is not whether students who are ELLs should learn vocabulary but rather how instruction can best support students to learn vocabulary *as they actively engage in mathematical reasoning about important mathematical topics*. Therefore, these materials and recommendations stress the importance of creating (and supporting students) in engaging in rich mathematical discussions.

The annotated tasks provided here all require that teachers develop skills and strategies for leading, supporting, and orchestrating mathematical discussions, whether these occur in small groups or with the whole class. These strategies are best learned in the context of a particular mathematical topic—for example, learning what the best questions are to support algebraic thinking (Driscoll, 1999), or geometric thinking (Driscoll, 2007). These strategies are also best learned through long-term professional development that engages teachers in observation, watching video, sharing lessons, etc. These skills for teaching mathematics are fundamental to supporting students in achieving the expectations set by the CCSS and are essential for supporting ELLs. Therefore, in Appendix A, we provide pointers to materials (books, videos, etc.) that can be used to support teachers in learning to orchestrate mathematical discussions.

REFERENCES FOR PREFACE

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6–10.* Portsmouth, NH: Heinemann.

Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5–10*. Portsmouth, NH: Heinemann.

Halliday, M. A. K. (1978). Sociolinguistics aspects of mathematical education. In M. Halliday (Ed.), *The social interpretation of language and meaning* (pp. 194–204). London: University Park Press.

Moschkovich, J. N. (2007). Examining mathematical discourse practices. *For The Learning of Mathematics, 27*(1), 24–30.

Moschkovich, J. N. (Ed.) (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age Publishing.

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WHAT HAVE WE DONE, HOW, AND WHY?

We began with mathematics tasks developed by the Mathematics Assessment Resource Service's Mathematics Assessment Project, and Inside Mathematics.

The goal was not to reduce language demands by altering the mathematics tasks but instead to provide support and scaffolding for ELLs to learn how to manage complex text in mathematics. There are several reasons not to adapt the texts of a task:

- Changing the language of a task can change the mathematical sense of the task.
- It is not yet clear which adaptations are best to make for which students, for which purposes, or at which times.
- Instruction should support students in understanding complex mathematical texts because they are likely to appear in curriculum and assessment materials.
- Experiences that allow ELLs to engage (with support) with authentic language used in mathematics can provide opportunities for their continued language development.

We then produced three types of resources to support teachers in learning to use CCSS-aligned mathematics tasks with ELLs:

1. Mathematics Tasks with Annotations, describing how to use a CCSS-aligned mathematics task with ELLs. (These are available as separate documents on the Understanding Language website.)

2. Pointers to Professional Development Materials (books, videos, etc.) that can be used by teachers to learn to orchestrate mathematical discussions. (This is Appendix A of this document.)

3. Templates for Language of Mathematics Tasks provides templates for five language-focused tasks. Teachers can use these templates to design and write their own Language of Mathematics tasks to fit a mathematics task of their choice. (This is Appendix D of this document.)

These resources were developed using the "Key principles for mathematics instruction for ELLs" (pages 9-15 of this document) and the "Guidelines for design of mathematics instructional materials for ELLS" (pages 16-22 of this document).

Descriptions of Resources

There are three types of resources:

1. **Mathematics Tasks with Annotations**
2. **Pointers to Professional Development Materials**
3. **Language of Mathematics Task Templates**

These resources are provided for teachers as exemplars of the type of instruction that supports mathematical reasoning and sense making using complex, rigorous, and academically challenging tasks or lessons for all students, including ELLs. Teachers are encouraged to generate their own lessons using these resources.

1. Mathematics Tasks with Annotations (on UL website)

Elementary School | *Roger's Rabbits*

Middle School | *Making Matchsticks*

High School | *Sidewalk Patterns*

High School | *Creating Equations*

Each mathematics task presents opportunities for students to develop skills called for by the Standards for Mathematical Practice. In addition, each task provides grade-level–appropriate opportunities for students to comprehend and produce mathematical language. They have a variety of structures: some are scaffolded, allowing students to familiarize themselves with a problem situation before being asked to perform tasks of higher cognitive demand, while one task presents students with an open-ended problem and asks them to collaborate in order to reach a reasonable consensus solution.

The annotations describe the standards afforded by each task. Each annotation includes comments and suggestions for how to use the task with ELLs, as well as "Language of Mathematics" tasks designed to support ELLs. For each task, annotations provide the following information:

- Core mathematical ideas in the task.
- CCSS for Mathematical Content, CCSS for Mathematical Practice, and CCSS for ELA/Literacy.
- Comments on the pedagogical purposes of the task.
- Suggestions for using the task with ELLs.
- Language of Mathematics tasks (one or more) with teacher directions and student materials.

2. Pointers to Professional Development Materials (Appendix A)

A central skill in teaching ELLs mathematics is supporting mathematical discussions in the classroom. This aspect of teaching mathematics is fundamental to teaching mathematics for understanding, supporting students in the CCSS, and engaging students in the mathematical practices. It is also essential for supporting ELLs to develop both mathematical proficiency and language. There are resources already available that can support teachers in developing these skills. Therefore, we provide Pointers to Professional Development Materials (books, videos, etc.) that can be used by teachers to learn to orchestrate mathematical discussions. Although teachers can read the materials on their own, the best settings for this type of professional development would be long-term study groups or professional development experiences.

3. Language of Mathematics Task Templates (Appendix D)

These templates are general descriptions for five language focused activities. Teachers can use these templates to write their own Language of Mathematics Tasks to fit a mathematics task of their choice.

The Language of Mathematics tasks focus on two issues, reading mathematics problems and using vocabulary, not because we think that these are the most central for learning mathematics *or* language but, instead, for several pragmatic reasons: a) These two issues are often raised by practitioners as the major stumbling blocks they face when teaching ELLs, and b) There were exemplars of tasks addressing these two issues which had been already piloted used with teachers. Descriptions for these activities and student materials were either: adapted from the Understanding Language, English Language Arts Unit, adapted from materials already used by teacher professional development professionals (R. Santa Cruz and H. Asturias), or recommended as "good bets" by researchers with expertise in how to scaffold vocabulary and support reading comprehension. The materials draw, in large part, on papers prepared for the Spring 2012 Understanding Language Conference at Stanford University (http://ell.stanford.edu/papers/practice) and the ELA unit written by Walqui, Koelsch, & Schmida (http://ell.stanford.edu/teaching_resources/ela).

Understanding Language

Language, Literacy, and Learning in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet Common Core State Standards and Next Generation Science Standards http://ell.stanford.edu.