Sidewalk Patterns

Level: High School

Version 8.11.13

This task gives students an opportunity to:

- Work with expressions, equations, and functions

Table of Contents

These Language of Mathematics tasks were designed to support students in learning to read word problems and talk about the mathematics. They are accompanied by suggestions for classroom use.

Name ______________________________

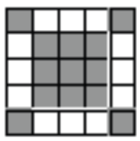
Sidewalk Patterns

This problem gives you the chance to:
- work with expressions, equations, and functions.

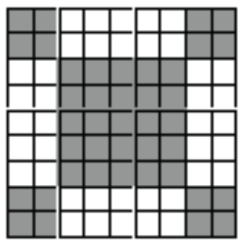
In Prague, some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns of various sizes.

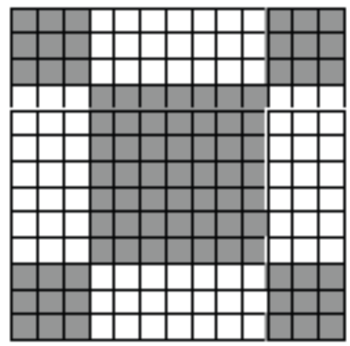
This problem is about how the patterns below are related.



Pattern number 1



Pattern number 2



Pattern number 3

1. How can Pattern 1 be changed to make Pattern 2?

   How can Pattern 2 be changed to make Pattern 3?

   Use words, diagrams, or symbols to describe these changes.

Adapted from the Noyce Foundation's Sidewalk Patterns.

2. Complete the table below by writing an arithmetic expression for each number of blocks. Some examples are in the table.

	Pattern 1	Pattern 2	Pattern 3
Number of white blocks	4×3	40	
Number of gray blocks	$(1 + 1 + 1 + 1) + 3^2$		
Total number of blocks	25		

3. In the completed table, there are *three* different arithmetic expressions for the number of white blocks in Patterns 1, 2, and 3.

Write *one* symbolic expression that represents the number of white blocks in each pattern.

4. In the completed table, there are *three* different arithmetic expressions for the number of gray blocks in Patterns 1, 2, and 3.

Write *one* symbolic expression that represents the number of gray blocks in each pattern.

5. In the completed table, there are *three* different arithmetic expressions for the total number of blocks in Patterns 1, 2, and 3.

Write *one* symbolic expression that represents the total number of blocks in each pattern.

Sidewalk Patterns | Annotations

Core Ideas

Sidewalk Patterns presents students with a sequence of three visual patterns created using white and gray blocks, and asks them to write one symbolic expression for the number of each type of blocks. It provides opportunities for work related to the standards listed below.

Note that a finite sequence does not determine exactly one pattern without additional constraints. Because of this, the Common Core State Standards do not require students to infer or guess a single underlying rule for a pattern when given a finite sequence.

Common Core State Standards for Mathematical Content

http://www.corestandards.org/Math/Content/HSA/CED

High School, Functions, Building Functions (p. 70)

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, and use them to model situations and translate between the two forms.

High School, Algebra, Seeing Structure in Expressions (p. 64)

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different

SMP.3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students . . . are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. . . .

SMP.4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP.6. Attend to precision.

Mathematically proficient students look closely to discern a pattern or structure. . . . They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. . . .

SMP.7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. . . . They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. . . .

SMP.8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. . . .

Common Core State Standards for ELA/Literacy
http://www.corestandards.org/ELA-Literacy

Grades 9–10, Writing (p. 45)

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. (Use precise language and domain-specific vocabulary to manage the complexity of the topic.)

Grades 9–10, Speaking and Listening, Comprehension and Collaboration (p. 50)

1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally), evaluating the credibility and accuracy of each source.

Grades 9–10, Reading for Informational Text, Craft and Structure (p. 62)

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9–10 texts and topics.

Grades 9–10, Reading for Informational Text, Integration of Knowledge and Ideas (p. 62)

7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

Comments

Purpose of the task. This task presents substantial opportunities for ELLs to directly engage in the mathematics, present their thinking about the mathematics, develop verbal explanations of their solutions, and develop explanations of their solutions in writing.

The nature of the task invites a wide range of approaches, strategies, and representations to come into play as students move from cases to generalizations, and from visual to numerical to algebraic expressions. This range presents a dynamic set of resources for ELL students to connect their own thinking to the language used by their peers, and for all students to build deeper and clearer understandings of the mathematics.

When working with ELLs, focus on supporting verbal and written communication, including creating space within the task for students to refine explanations of their strategies and generalizations.

Structure for framing the task. Part of the work of the task for all students must be to make connections among distinct approaches, strategies, and representations, including:

- identifying algebraic expressions that are equivalent.
- developing and clarifying approaches that are partial or incomplete.
- making generalizations from cases.
- identifying strategies that appear distinct but are algebraically equivalent.

Using the variety of ideas generated by this task as a resource for all students would support SMP.1 ("Make sense of problems and persevere in solving them"), SMP.7 ("Look for and make use of structure"), SMP.8 ("Look for and express regularity in repeated reasoning"), and SMP.3 ("Construct viable arguments and critique the reasoning of others").

Language of Mathematics tasks. The last section of this document provides handouts and teaching suggestions for two Language of Mathematics tasks. These were designed to support students in learning to read word problems and talk about their solutions and reasoning. The tasks are:

Reading and Understanding Sidewalk Patterns (pages 17–18). This provides a structure for students to work in pairs or small groups. The directions support students as they read and explain each question in this task.

Mathematically Speaking: Sidewalk Patterns Poster Presentation (page 19). This provides a structure for students to describe their solutions both orally and in writing (available on the Understanding Language web site).

These Language of Mathematics tasks are provided as resources to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Focus students on mathematical reasoning. As students work in groups, when they make presentations, and during whole class discussions, ask them to describe both orally and in writing:

- What they did to solve a problem or find an answer
- Why they did that step, and
- Why that step is justified mathematically; for example, asking "What is the mathematical reason for that step?"

Suggestions

Task as a whole. A likely hurdle for ELLs (and other students) is identifying and naming salient quantities: total number of blocks, number of gray blocks, number of white blocks, and possibly others.

- Language development in mathematics requires students to use language in talking, listening, reading, and writing. This is where paired work is essential so each student talks and is listened to.
- While working with pairs or individual students directly, refer to strategies used by other pairs or groups of students.
- Allow a variety of approaches to take shape. then stop pair conversations to have the whole class share out.

Before Question 1

- Before students work on Question 1, ask them to work alone, look at the first two patterns and write down responses to these questions: "What is the same for both patterns? How are the two patterns different?" Then ask students to share their responses with a partner.
- After the groups work on the question, have a few groups share their findings. The variety of oral explanations will create opportunities for connections between oral and written descriptions.

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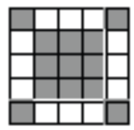
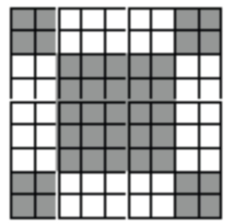
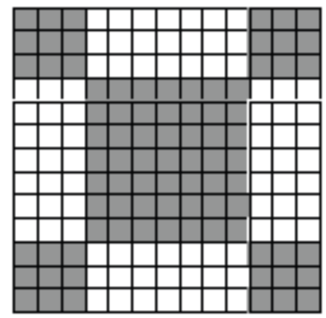


Pattern number 1



Pattern number 2



Pattern number 3

1. How can Pattern 1 be changed to make Pattern 2?

   How can Pattern 2 be changed to make Pattern 3?

   Use words, diagrams, or symbols to describe these changes.

Question 1

Students may move too quickly without understanding each other's reasoning or developing their own reasoning.

- Ask students to fill in the table together in pairs. While working in pairs, they should check each other's work and agree before going on.
- Go over the table as a class and make sure everyone agrees on the values of the expressions, before they go on. This is an opportunity for students to identify equivalent and non-equivalent arithmetic expressions before they use algebraic expressions for Question 2. It is also an opportunity for students to explain correspondences between terms in arithmetic expressions and areas in diagrams. For example, in Pattern 1 the 3^2 ("3 squared") in the table represents the area of a 3 by 3 square and each of the four 3s represents the area of a 1 by 3 rectangle.

Question 2

2. Complete the table below by writing an arithmetic expression for each number of blocks. Some examples are in the table.

	Pattern 1	Pattern 2	Pattern 3
Number of white blocks	4×3	40	
Number of gray blocks	$(1 + 1 + 1 + 1) + 3^2$		
Total number of blocks	25		

The question may need clarification.

Ask pairs to:

- Make an observation in words (verbally) to their partner about how the number of white blocks in Pattern 1 compares with the number of white blocks in Patterns 2 and 3.
- Write down what their partner said.
- Read what they wrote to their partner, and vice versa.
- Refine or synthesize each observation as needed.

Have one partner to express the relationship between the numbers of white blocks in the patterns algebraically, with expressions using letters, then have the other partner translate the expressions into words. Next, the pair revises the expressions as needed.

Allow a variety of approaches to take shape, then stop pair conversations to have the whole class share out. Agree on one or two approaches, e.g., using *n* to represent the number of the pattern and using *s* for the side length of any pattern.

Review as a class several examples of what students are expressing about what they notice. This will give everyone a chance to “see and hear” the reasoning used by their peers and to make their constructions visible on the board on a larger scale.

This process can repeated after students work on Question 4 and again after Question 5.

LANGUAGE OF MATHEMATICS TASKS

Two Language of Mathematics tasks designed to work together

- Reading and Understanding Sidewalk Patterns
- Mathematically Speaking: Sidewalk Patterns Poster Presentation.

Reading and Understanding handout adapted from handout developed by Harold Asturias.

Mathematically Speaking handout adapted from the work of R. Santa Cruz (2012) for the Understanding Language Project.

Purpose

Reading and Understanding Sidewalk Patterns. The purpose of this task is to support students in learning to read, understand, and extract relevant information from a mathematics problem and, in doing so, to develop an approach to solving the problem.

Mathematically Speaking: Sidewalk Patterns Poster Presentation. The purpose of this task is to support students by providing a structure for them to describe their solutions by first working alone, then working in pairs to describe their solutions both orally and in writing, and finally making a poster presentation. The tally sheet on page 19 is designed to be used during the presentations, but can also be revised to support students as they prepare their presentations.

Required for use

- Reading and Understanding handout: one copy for each pair of students.
- Square tiles or grid paper, poster paper, and markers.
- Mathematically Speaking tally sheet with target vocabulary: one copy for each pair of students.

Structure of the activity

- Students begin by reading or attempting to read the problem individually, and then immediately form pairs to talk through the problem together using the handout provided. There are four steps in talking through the problem together, three of which begin with reading the problem aloud.

- The *first step* involves identifying what the problem is about (e.g., sidewalks made of small square blocks of stone, different shaded blocks, patterns formed using the square blocks).
- The *second step* involves asking what information students are supposed to find (look for patterns to find the number of white blocks and gray blocks for each pattern). Students answer these questions together, both orally and in writing.
- The *third step* involves a scaffolded set of questions leading to descriptions of relationships between the number of the patterns and a given quantity in the pattern: number of white blocks, number of gray blocks, total number of blocks.
- The *fourth step* is for students to make a poster (in groups of four) to show how they arrived at a symbolic expression for the numbers of blocks in each pattern. Students must show that their expression works for each pattern in the sequence, and use multiple representations to explain their thinking (using diagrams, tables, words, and expressions and equations). Poster paper and grid paper are provided.

Before asking students to present to the whole class, provide students an opportunity in the small groups to think about, write, and practice orally how to use the target vocabulary, using the handout "Mathematically Speaking" on page 19.

- Groups present their work to the class using the target vocabulary of the lesson (e.g., *pattern, sequence, term, total number, squaring, expression*). Where necessary, the teacher will pose questions to elicit the appropriate vocabulary for discussing the patterns in the sequence of figures.
- As groups present their work, the class will tally the number of times the target vocabulary of the lesson is used.

Process outline

- Students work individually on the problem.
- Students form pairs and begin to talk through the problem, sharing one copy of the Reading and Understanding handout.
- Pairs of students talk together to answer the questions in Steps 1–3 on the Reading and Understanding handout, both orally and in writing.

- Finally students work in groups of four to make a poster, showing how they arrived at a solution. They explain their thinking using multiple representations: words, expressions and equations, tables, diagrams, first to the group, then to the whole class. During the presentation to the class, listening students tally word use on the Mathematically Speaking handout (page 19).

Reading and Understanding Sidewalk Patterns

Names: ______________________ and ______________________

Step 1. Read the problem out loud to a peer. Try to answer these questions.

What's the problem about?

Step 2. Read the problem again.

What are the questions in the problem? What are you looking for?

Step 3. Read the problem a third time. Talk to your partner about these questions.

a. What do you observe as you go from one pattern to another? What is changing and what is staying the same? Talk about the pattern you see in the white blocks and the gray blocks.

b. How can you find the number of white blocks and gray blocks in each pattern, along with the total number of blocks?

Reading and Understanding Sidewalk Patterns

Names: ________________________ and ________________________

c. What is a relationship between the number of white blocks in each pattern and the number of the pattern (1, 2, or 3)?

d. What is a relationship between the number of gray blocks in each pattern and the number of the pattern (1, 2, or 3)?

e. What is a relationship between the total number of blocks in each pattern and the number of the pattern (1, 2, or 3)?

Step 4. Work in groups of four (4) to make a poster showing how you arrived at a symbolic expression to find the total number of blocks in each pattern. Show that your expression works for each pattern in the sequence and use multiple representations to explain your thinking (diagrams, tables, words, and expressions or equations). Poster paper and grid paper are provided.

Create an oral presentation of your ideas using the target vocabulary on the handout "Mathematically Speaking" (next page). When you present your ideas to the whole class, you will be expected to use these words. Ask the teacher for help if you do not understand how to use the vocabulary.

As your group presents its work, the class will tally the number of times the target vocabulary of the lesson is used. A chart is provided on the handout "Mathematically Speaking" (next page).

Mathematically Speaking: Sidewalk Patterns Poster Presentation

Names: ___________________________ and ___________________________

For each group's presentation, mark a tally on the chart every time you hear the presenter use one of the target vocabulary words.

Add to the chart any words you hear that are important in the presentations.

Target Words	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Pattern						
Sequence						
Term						
n^{th} term						
Total number						
Squaring						
Squares						
Rectangles						
White blocks						
Gray blocks						
Arithmetic expression						
Symbolic expression						

Understanding Language | Language, Literacy, and Learning in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet the Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS).

http://ell.stanford.edu