Making Matchsticks

Level: Middle School

Version: 7.2.13

This task is intended to help teachers assess how well students are able to:

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods.
- Interpret and evaluate the data generated.
- Communicate their reasoning clearly.

It is accompanied by a formative assessment lesson.

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This Language of Mathematics task was designed to support ELL students in learning to talk about the mathematics in Making Matchsticks. It is accompanied by suggestions for classroom use.

Name _____

Making Matchsticks

Matchsticks are rectangular prisms of wood measuring approximately:

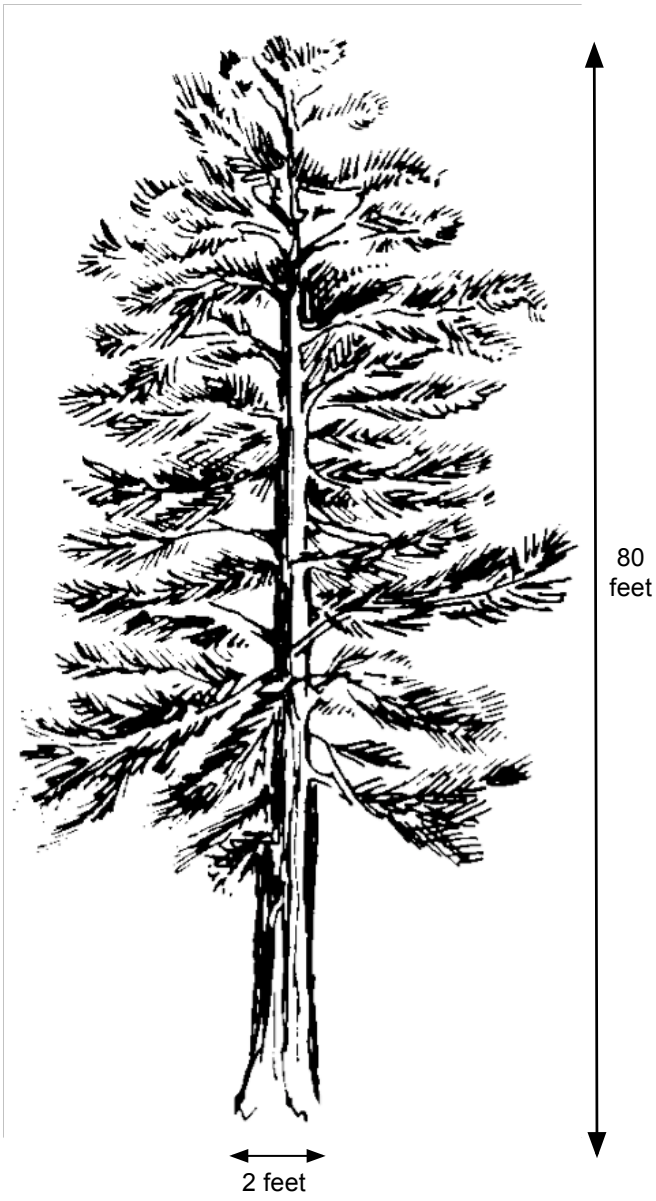
$\frac{1}{10}$ inch by $\frac{1}{10}$ inch by 2 inches

Matchsticks are often made from pine trees.

Estimate how many matchsticks can be made from this tree.

You may find some of the information given on the formula sheet helpful.

Explain your work carefully, giving reasons for any choices you make.



Making Matchsticks | Annotations

Core Idea

This task and the associated formative assessment lesson afford opportunities for students to develop strategies for solving problems. Students select formulas from a sheet of geometric formulas in order to model a situation. They interpret given data, make approximations, and communicate their reasoning in verbal and written form. Students analyze and critique solutions developed by others.

Note that although the task was designed to have students select volume formulas from a list, students are expected to know these formulas by the end of grade 8. This task draws on understandings of rate and proportional reasoning (a focus of grades 6 and 7), geometric measurement and volume which begins with right rectangular prisms in grade 5, and is extended in grades 6, 7, and 8. It is an opportunity to build toward the high school number and quantity standard of interpreting units consistently in formulas.

Common Core State Standards for Mathematics
http://www.corestandards.org/Math/Content/8/G

Grade 8, Geometry (p. 56)

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. . . .

SMP.3. Construct viable arguments and critique the reasoning of others.	Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .
SMP.4. Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
SMP.6. Attend to precision.	Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .

Common Core State Standards for ELA/Literacy
http://www.corestandards.org/ELA-Literacy

Grade 6, Writing, Text Types and Purposes (p. 42)	1. Write arguments to support claims with clear reasons and relevant evidence.
Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)	2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.
Grade 6, Speaking & Listening, Presentation of Knowledge and Ideas (p. 49)	5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.

Grade 6, Reading Standards for Informational Text, Integration of Knowledge and Ideas (p. 39)

7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.

Comments

Purpose of the task. Making Matchsticks is part of a MARS Formative Assessment Lesson (FAL), which can be downloaded here: http://map.mathshell.org/materials/lessons.php?gradeid=23.

In these FALs, initial individual student work on the task should not be supported or scaffolded by the teacher because their work is intended to produce evidence of their current knowledge. This assessment of students' prior knowledge will be used as the basis for the lesson.

Using the task with ELLs. Although students' mathematical work should not be supported or scaffolded, teachers will gain significantly more information about students' prior knowledge of the mathematics needed for the task if students comprehend what they are being asked to do in the task. In other words, although an unsupported attempt at the task by an ELL student will generate evidence of how well the student understood the task instructions, the attempt is less likely to inform a teacher about the mathematics the student would bring to the task with a clear understanding of what was being asked. Because of this, even if the task is to be used for individual assessment, we recommend that the teacher provide some support for ELLs for the individual work on the initial task for MARS Formative Assessment Lessons. The sole purpose of this support should be to give students access to the task—not directly to the mathematics they need to do the task, but rather to a clear understanding of what they are being asked to do.

Outline of the lesson. The MARS student materials include the task, a formula sheet, and a questionnaire called How Did You Work? After students have worked on the task individually (in class or as homework), they discuss the task and various responses (as described in detail in the MARS materials). Each small group of students shares a large sheet of paper for making a poster and copies of sample responses.

Here is the outline as given in the MARS materials:

1. Before the lesson, students tackle the problem individually. You assess their responses and formulate questions that will prompt them to review their work.

2. At the start of the lesson, students respond individually to the questions set, and then work in groups to combine their thinking and produce a collaborative solution in the form of a poster.

3. In the same small groups, students evaluate and comment on some sample responses. They identify the strengths and mistakes in these responses and compare them with their own work.

4. In a whole-class discussion, students explain and compare the solution strategies they have seen and used.

5. Finally, students reflect on their work and their learning.

The suggestions and the Mathematically Speaking task in these annotations are intended to supplement the lesson described in this outline. The suggestions supplement parts 1, 2, 3, and 5. The Mathematically Speaking task supplements the poster presentations in part 4.

Some potential challenges in the task are that students might:

- not take into consideration the different units used for toothpick and tree measurements.
- not understand and apply the various formulas for volume.
- misplace the decimal.
- envision the tree as two-dimensional.

For more potential challenges, see the table of possible responses in the next section.

Suggestions

The minimal support we recommend for using Making Matchsticks as an individual assessment is to ask students to mark the task, replacing "inch" by "in" and "80 feet" by "80 ft," and to identify the objects on the upper right as matchsticks. This support is intended only to present the question and the given information more clearly, and not to add information or suggest a solution path.

Before students work on Making Matchsticks. Instructions from the teacher should be clear, direct, and concise. The direction in this activity might be reduced to the following:

- Estimate how many matches can be made from the wood in this tree.
- Use the relevant information on the formula sheet. It will help you find some answers.

- Read the task, and show all your work. Showing your work helps me understand your reasoning (thinking).
- It is important that your work is organized and presented in a clear manner (way).

Before the lesson: Assessing students' responses on Making Matchsticks. The Common Issues table below is copied and pasted from the online teacher materials for the MARS lesson Making Matchsticks; additional suggestions and prompts from the Understanding Language Project are given below this table.

Common issues:	Suggested questions and prompts:
Student has difficulty getting started	• What do you know? What do you need to find out? • How could you simplify the problem?
Student ignores the units For example: The student calculates the volume of a matchstick in cubic inches and the volume of the tree trunk in cubic feet.	• What measurements are given? • Does your answer seem reasonable if you consider the size of a matchstick compared to the size of a pine tree?
Students makes incorrect assumptions For example: The student multiplies the volume of the tree trunk in cubic feet by 12 and assumes this gives the volume of the tree trunk in cubic inches.	• Can you explain why you have multiplied by 12? • When you figure out a volume how many dimensions do you multiply together? How does this calculation effect how you convert the volume from cubic feet to cubic inches? • Can you describe the dimensions of the tree in inches? What do you notice?
Student uses an inappropriate formula For example: The student calculates the *surface area* of a rectangular prism from the dimensions given for the tree.	• Does your choice of formula make good use of all the wood in the tree trunk? • Is this the best model for a tree trunk? • What is the difference between area and volume?
Students' work is unsystematic	• Would someone in your class who has not used this method be able to follow your work? • Can you describe your method as a series of logical steps?
Students' work is poorly presented For example: The student underlines numbers and it is left to the reader to work out why this is the answer as opposed to any other calculation.	• Can you explain each part of your solution? • What does each of these calculations represent? • Can you justify the choices you have made?
Student has difficulties when substituting into a formula For example: The student multiplies the radius by 2, rather than squaring, when using the formula for the volume of a cone/cylinder. Or: The student substitutes diameter rather than radius into the formula for the volume of a cone/cylinder.	• What is the difference in meaning between $2r$ and r^2? • Does your answer seem reasonable? • How can you check your work against the information given in the problem?

Students' work is incomplete For example: The student does not divide the volume of the tree trunk by the volume of a matchstick.	• What do your calculations represent? • Have you found out how many matchsticks can be made from the tree?
Student rounds to one or more decimal places	• Why won't part of a matchstick count in your estimate?
Student completes the task	• How can you check that the method you have used has given a reasonable estimate? • Can you try a different method to check your answer? • What assumptions have you made?

Below are additional issues not found in the existing Common Issues table for this lesson, together with suggested questions and prompts. Note that these provide considerably more scaffolding than the prompts above.

Student attempts to use proportional reasoning to develop a solution path but gets confused.	• What is the rate of inches per foot? • What is the rate of cubic inches per cubic foot? • How can you use these rates to explain your solution to the problem?
Student struggles to make sense of linear, square, and cubic units.	• Draw three figures: first, draw a length of 1 inch; second draw a square with 1 inch side lengths; third, draw a cube with 1 inch side lengths. Then carefully label the side lengths in each drawing. • (If a student does not understand the exercise above, model each drawing with 1 inch, then ask the student to do the exercise with 1 foot. Then ask how many linear inches are in each linear foot, how many square inches are in each square foot, and how many cubic inches are in each cubic foot.)
Student has trouble getting started. **(Note:** this is the entry in the first row of the existing table; to the right is an additional prompt for this entry.)	• Re-read the problem aloud to your partner(s).
Student ignores the units. **(Note:** This is the entry in the second row of the existing table; to the right are an additional prompt and question that belong between the two existing questions.)	• What is being measured in this problem? • Try to explain to your partner(s) how the measurements you are given can help you find the measurements you need in order to solve the problem.

Beginning of the lesson: Reviewing individual solutions. Include a printed list of questions on which students can reply.

Some possible sentence starters for review process:

- At first I thought . . .
- I realized that . . .
- However . . .

- Now, I see that . . .
- Next time, I will . . .

Whole class discussion after students have discussed sample approaches. Possible questions to ask:

- Which approach or paper made the most sense to you? (Like and dislike may not be the best language to use here.) In which way(s)?
- Which approach was difficult to understand? Why?

At the end of the lesson or for homework. Possible questions to ask:

- Think about the different methods we used in this lesson.
- What was the most important thing you learned from your work today?
- Write about one thing that you are wondering about, find unclear, or need help with.
- Share your writing with another student.

Language of Mathematics Task Mathematically Speaking

Adapted from R. Santa Cruz (2012) for the Understanding Language Project

This task is to be used with the Formative Assessment Lesson associated with Making Matchsticks during group creation and presentations of posters.

The Language of Mathematics task is provided as a resource to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Purpose

This task gives students practice tracking and interpreting target vocabulary words used by their peers during prepared group presentations. Also, it supports students in producing language, since students will be encouraged to prepare and give presentations that incorporate correct use of target words. It is not intended to give students access to the mathematics of the task during the central work on the problem, but rather to provide opportunities for using and understanding key terms when students summarize their work during poster presentations after the central work is complete.

It is crucial that students do this vocabulary work *after* they solve the problem that grounds the meanings for words. Students are likely to use everyday words in their talk while solving a problem in groups and should not be corrected. Instead, the teacher can provide guidance on more formal mathematical terms during a whole-class discussion by providing instruction on the mathematics while modeling the use of more formal vocabulary and ways of talking.

Developing academic language is more than just learning the target or specialized vocabulary of a unit or chapter. Comparative structures

such as "one third of," "at least,'" or "twelve times the amount" are the kinds of syntactic structures that students need to understand and use as they describe their work or make a presentation about their solution for a mathematics task.

Required for use

- Materials for MARS lesson Making Matchsticks.
- Mathematically Speaking tally charts with target vocabulary words.

Process details

1. Before using the Mathematically Speaking tally chart, students spend a couple of minutes reviewing their individual solutions to Making Matchsticks (completed in advance of the lesson). Use the following prompts to help students prepare to work with their peers:

    - Summarize the steps that you took to solve the problem.
    - What part of your solution are you most sure about? What part are you least sure about?

    Students can share their responses with their peers and reach a consensus as to the "best" solution to the problem. Together, students create posters displaying this joint solution to the problem.

2. After the pairs or groups create the posters, their members present to each other or to the same group (e.g., one member of a pair or group presents to the other members) using the tally sheet to help each other insert the academic language appropriately. The use of recording devices (e.g. iPods, iPads, tape recorders) can help facilitate this process as students can initially record what they want to say and replay their practice presentations. This provides a "safer" way for students to learn how to use academic language and helps ensure student success when they present in front of the whole class.

1. When pairs or groups are finished with their posters, they describe their posters to the whole class. Each student gets one copy of the chart with target vocabulary words entered. As each group describes their poster, each listening student tallies on the chart each time a target word is used. If a target word is not used, the listening students should encourage the presenters to keep

talking, by asking questions or requesting clarifications, until all target words on the list have been used.

2. Students may add words to their chart that come up in their explanations, and then share these with the class at the end of the activity.

3. The lesson continues as described in the MARS lesson outline, with the collaborative analysis of sample student responses.

Process outline

- Each student receives a copy of the handout with the tally chart.
- Each group presents their poster, while other students mark each use of the target vocabulary words on the tally chart, and add words as needed.
- Students are asked to read the list of targeted words silently before the presenters start.
- Stress how important it is for the listeners to pay attention to presenters and listen for targeted words.
- Groups begin their presentations.
- At the end of all presentations, students go over any words they may have added to the given list.

Mathematically Speaking

Name _____ Date _____

For each group presentation, mark a tally on the chart every time you hear presenters use one of the target vocabulary words.

Add any words to the chart you hear that are important in the presentations.

Target Word(s)	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Approximate						
Convert						
Cube/Cubic						
Diameter						
Estimate						
Feet						
Formula						
Height						
Inches						
Length						
Measure						
Measurement						
Radius						
Rectangular prism						
Round						
Units						
Volume						
Width						

Understanding Language

Language, Literacy, and Learning in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet the Common Core State Standards (CCSS) and Next Generation Science Standards.

http://ell.stanford.edu