Creating Equations

Level: High School
Version 8.11.13

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Student Task

Annotations

Language of Mathematics Tasks

These Language of Mathematics tasks were designed to support students in learning to read word problems and talk about the mathematics. They are accompanied by suggestions for classroom use.

Creating Equations

1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables

2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

2b. Dave sold 40 tickets for a concert. He sold x tickets at \$2 each and y tickets at \$3 each. He collected \$88.

Write two equations connecting x and y.

Solve these two equations to find how many of each kind of ticket he sold.

2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2.

Write a quadratic equation, and solve it to find the length and width of this rectangle.

Adapted from Creating Equations 1 © MARS University of Nottingham

CREATING EQUATIONS | ANNOTATIONS

Core Idea

This task affords students opportunities to use algebra in different ways: manipulating a given equation, writing equations to represent situations, solving equations, and interpreting solutions of equations in the situations represented.

Common Core State Standards for Mathematical Content
http://www.corestandards.org/Math/Content/HSA/CED

HIgh School, Algebra. Creating Equations (p. 65)

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities. . . .

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

HIgh School, Algebra. Reasoning with Equations and Inequalities (p. 65)

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. . . .

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. . . .

SMP.2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. . . .

SMP.3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .

SMP.4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams . . . and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP.6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .

Common Core State Standards for ELA/Literacy
http://www.corestandards.org/ELA-Literacy

Writing Standards, Grade 9–10 (p. 45)

2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

Speaking and Listening Standards, Grades 9–10 (p. 50)

1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

Reading Standards for Literacy in Science and Technical Subjects, grade 9–10 (p. 62)

7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11–12 texts and topics.

Comments

Purpose of the task. Creating Equations provides opportunities for students to create and use algebraic equations. The task has four questions. Students are given a literal equation with more than two variables (Question 1) and asked to solve for one of the variables, asked to solve a word problem with one variable (Question 2a), and asked to solve word problems that involve simultaneous linear equations in two unknowns (Questions 2b and 2c). Although the different questions are unrelated in context, they are related in mathematical content.

Language of Mathematics tasks. The last section of these annotations provides three Language of Mathematics tasks. These tasks were designed to support students in learning to read word problems and talk or write about their solutions and reasoning. They are:

Jigsaw Reading for Creating Equations Question 2a (pages 14-20)

Reading and Understanding: Creating Equations 1 (pages 21-24)

Mathematically Speaking: Creating Equations 1 (pages 25-29)

Jigsaw Reading provides one way to support students in learning to see the structure of the word problem in Question 2a. The version of Jigsaw Reading provided here fits Question 2a, but the task structure can be revised to fit other word problems with longer texts. It is possible that Jigsaw Reading may work better for word problems that have more than three sentences.

Reading and Understanding: Creating Equations provides a structure for students to work in pairs or small groups. The directions support students as they read and explain each question in this task. The version provided here is designed for the three parts of Question 2, but can be revised to fit Question 1.

Mathematically Speaking provides a structure for students to learn to describe their solutions by first working alone and

then working in pairs to describe their solutions both orally and in writing.

These Language of Mathematics tasks are provided as resources to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with these questions themselves. Students first work alone, then in pairs or groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Suggestions

Question 1

1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables

Introducing the question: Whole class and/or small groups

The teacher can introduce the question by:

1. Giving a simple example of an equation using speed, distance, and time or other familiar quantities recently studied.

2. Asking students to discuss the meaning of each question in small groups and to restate the question in their own words. Some possible responses include:

   - In Question 1, *V* is expressed in terms of the variables *r* and *R.* What would the equation look like if *R* was expressed in terms of the variables *V* and *r?*

   - The equation in Question 1 gives *V* in terms of *R* and *r.* Write an equation that gives *R* in terms of *V* and *r.*

   - If *V* is equal to 12*R* divided by the quantity (*r* + *R*), then what would the equation be if we wanted to say *R is equal to* _____*?*

   - *If V = 12R ÷ (r + R), then R = ?*

Small group work: Focus students on mathematical reasoning

1. Ask students to work in groups to "undo" the operations.

2. Ask students to describe both orally and in writing:

   - what they did to "undo" each operation.

   - why they did that step.

   - why that step is justified mathematically, asking "What is the mathematical reason for that step?"

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write and explain their group's solution on the board along with the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

Question 2a

See Jigsaw Reading (pages 14-20) for one way to support students in learning to see the structure of this word problem.

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small groups.

For Reading and Understanding, students first read the question or problem on their own, then with a partner or in a larger group. Next, students are asked to discuss what the problem is about and what quantities are being measured or counted. Next, students identify and list all numerical values, and, if possible, draw a diagram. Students can use examples with real marbles or other objects to illustrate for each statement several ways in which it might be satisfied.

Introduce the question

1. Ask students to examine the sentence "*Find how many marbles Maria has.*" Then ask students to describe how this sentence may be rephrased. One possible response is as the question, "*How many marbles does Maria have?*"

2. Remind students to state the meaning of the variables they use, e.g., "let x be the number of marbles Ben has."

3. Remind students that the variable does not always have to be represented by x, it can be represented by any letter.

4. Remind students that they can pick which quantity will be their variable, for example:

   • let b be the number of marbles Ben has, or

- let *m* be the number of marbles Maria has, or

- let *j* be the number of marbles Jane has.

Small group work: Focus students on mathematical reasoning

Ask students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board along with the reasons for each step. The whole

2b. Dave sold 40 tickets for a concert. He sold *x* tickets at $2 each and *y* tickets at $3 each. He collected $88.

Write two equations connecting *x* and *y*.

class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

Question 2b

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small group.

When working on Reading and Understanding, students:

1. Read question 2b on their own.

2. Read question 2b to their partner or to the group.

3. Discuss with each other what the problem is about and what quantity is being measured or counted.

4. Identify and list all numerical values, and, if possible, draw a diagram representing each statement. Students can use examples with objects to illustrate several ways that each statement might be satisfied.

Introduce question 2b

1. Point out the use of "collection of marbles" in the previous question (2a) and the use of "collected $88" in this question (2b).

2. Ask students how many variables they need for this question.

If students need more help, provide the following guidance:

- Write one equation that shows how many tickets Dave sold.

- Write another equation that shows the amount of money Dave received in ticket sales.

- Use both equations to find how many of each kind of ticket Dave sold.

Small group work: Focus students on mathematical reasoning

As students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board along with the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

Question 2c

2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2.

Write a quadratic equation, and solve it to find the length and width of this rectangle.

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small groups.

Introduce question 2c

1. Point out that "cm" means "centimeters."

2. Ask students to list all measurements mentioned in the problem.

3. Ask students to replace a pronoun with a noun by replacing "its" with "the area of the rectangle" and rephrasing the second sentence as "The area of the rectangle is 60 cm^2."

If students need additional guidance:

- Ask students to choose different values for *x* and compute the area of the rectangle in each case to make the computation for area explicit.

- Ask students to draw the figure described in the problem and to label the sides of the figure as *length* and *width*.

- Remind students to state what variables represent.

- Ask students if they remember any formulas associated with a rectangle. If they don't remember any formulas, ask them for the formula that gives the area of a rectangle.

- Clarify, explain, or rephrase the last sentence as "Write an equation showing how the product of the length and width for this rectangle gives you the area."

- Ask students whether the equation is a linear equation or a quadratic equation? Some students may need to expand the product before they can answer this question.

- Ask students "Why is it a quadratic equation?" Some students may need to expand the product before they can answer this.

Small group work: Focus students on mathematical reasoning

As students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

Ask students to:

- Solve the equation and use the solution to find the length and width of the rectangle.

- Check if the length and width found produces a rectangle with area 60 cm^2.

- Discuss why there are two solutions to the quadratic equation and why the negative solution does not yield a rectangle, thus does not satisfy the problem specifications.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board and describe the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

JIGSAW READING FOR QUESTION 2A

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).

Note: Jigsaw Reading is an activity in a very early draft form and has not yet been piloted in classrooms. It may turn out to work best with longer complex mathematics text.

Purpose

This Language of Mathematics task is intended to support students in learning to read and understand word problems. It asks students to think about how the sentences in a word problem are organized, how the text for a word problem flows, and how word problems are predictable. For example, the structure of a word problem typically begins with the statement of some given information, then more information, ending with a question or request for a solution or missing information. The text often includes words that are organizational markers, such as "find," "solve," or "write," or blanks that indicate that an answer is expected. The organization of paragraphs or sentences often follows the sequence: "given information," "more information," "request for solution."

The task requires that students carefully read parts of a word problem to determine where a particular sentence, written on a strip of paper, fits in relation to the other passages. In the process, students begin to focus, without prompting, on how selected grammatical and mathematical terms create cohesion and meaning within and across sentences, and how larger units of text are connected to create a word problem. The goal is to apprentice students into the type of close reading needed to understand more complex word problems and other mathematics texts.

Group structure and materials

- Groups of 3 students

- Copies of the text strips (one complete set of text strips for each group)

- Scissors

- Envelopes (one envelope for each group of students)

- A mathematics task or text that is no longer than a half page (in this case, Creating Equations Question 2b).

Initially, the text on the strips should contain clear organizational markers that are typical of the particular type of mathematics problem it illustrates. As students become more sophisticated readers of mathematics problems, they may benefit from reading and reassembling texts that are clearly organized but do not use understood markers to signal organization.

Structure of the activity

Initially, the teacher describes the overall purpose of the activity by explaining that writers use language to connect ideas within and across sentences and paragraphs in a word problem. In this activity, students will reassemble the text of a word problem whose sentences appear out of order on strips of paper to help students understand how sentences or paragraphs within a word problem are connected with each other. The teacher might introduce this activity with a mathematics task or text that is familiar to the class.

The text for Question 2a is partitioned into three strips (one set), with one sentence on each strip. A complete set of the three strips is placed in each envelope. The teacher distributes and reviews the directions on the student handout and gives each group an envelope of strips.

Process outline

Small group work

1. Describe to the class the directions below. Have students form groups of three.

2. One student distributes the text strips randomly to his or her group members.

3. Each student then reads his or her strip silently and decides where the text fits in the whole word problem: Is it the beginning? The middle? The end? Students must give reasons for their decisions.

4. When everyone in a group appears to be ready, the student who thinks he or she has the first text strip says, "I think I have the first strip because . . ." and, without reading the text aloud, explains their rationale. If any other group members think they have the first strip, they too must explain, "I think I have the first strip because . . ."

5. Once the group decides which student has the first strip, that student reads his or her strip aloud.

6. After hearing the text, the group decides whether it is indeed the first part of the problem. Once agreement is reached, the strip goes face up on the table where group members can refer to it as needed.

7. Students follow the same procedure to reconstruct the rest of the problem, strip by strip.

8. If students feel they have made a mistake along the way, they go back and repair it before continuing.

9. Once the whole process is finished, all group members review the "jig-sawed" text to make sure it has been assembled correctly.

Whole group discussion. The teacher can facilitate a whole group discussion asking students to explain which words, phrases, connectors, or other linguistic features helped them ascertain the order of the strips. The teacher can use strips of transparencies on the overhead (or document camera), or paper strips on a white board (or document camera) to manipulate during student explanations. When warranted, the teacher can provide different ways to state relationships given in the text for the word problem.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide a structure that allows students to grapple with the organization of a word problem by themselves.

The structure of the activity is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

Jigsaw Reading for Creating Equations Question 2a

> 2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

Each sentence of question 2a appears on one of three strips below. Cut along the dotted lines and place each collection of three strips in envelopes in a random order.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Name _____

Jigsaw Reading | Creating Equations Question 2a

Step 1. One person in your group distributes one strip of paper to each person in your group. Read your section silently and decide if your strip appears in the beginning, middle, or end. Use the box below to write your decision and the reason for your decision.

Write beginning, middle, or end in the blank for your strip and finish the sentence below.

I think I have the _____ strip because _____

_____.

Order	Reason for my decision

Step 2. Check to see if everyone is ready and begin.

1. The person who believes they have the beginning starts by saying, "I think I have the beginning because . . . "

2. If more than one person believes they have the beginning, the group must also hear their reasons. Talk within your group to reach agreement. Once your group agrees which person has the beginning, that person reads their text aloud and the strip goes face up on the table.

3. The person who believes they have the middle goes next by saying, "I think I have the middle because . . . "

4. If more than one person believes they have the middle, the group must hear their reasons. Talk within your group to reach an agreement. Once your group agrees which person has the middle, that person reads their text aloud and the strip goes face up on the table.

5. The person who believes they have the end says, "I think I have the end because . . . "

6. Once your group reaches agreement on the order, one person reads the entire problem aloud to the group.

7. Write the entire problem in the space below.

Step 3. Decide whether each strip has information that is known, information that is not known, and/or information that you want to know. In the space where the entire problem is written, write K, N, or W, on top of each part of the beginning, middle, and end strips to indicate the following:

K is for information that is Known.

N is for information that is Not known.

W is for information that you Want to know.

Organize these types of information with your group in the table below.

Information that is known:	
Information that is not known, but is needed to solve the problem:	
Information that you want to know to solve to solve the problem:	

READING AND UNDERSTANDING CREATING EQUATIONS

Adapted from work developed by Harold Asturias

Purpose

The purpose of this Language of Mathematics task is to support students in approaching a mathematics problem. It gives students tools for learning to read, understand, and extract relevant information from a problem, and gives them practice using this information to make inroads toward the additional information they are trying to find in order to solve the problem.

Required for use

- Reading and Understanding Creating Equations handout: three copies of the handout for each pair of students.

- Questions 2a, 2b, and 2c: one copy of the worksheet for each student.

Structure of the activity

1. Students begin by individually reading or attempting to read the problem (one of questions 2a, 2b, or 2c), then immediately form pairs to talk through the problem using the Reading and Understanding handout.

2. There are four steps in talking through the problem together, three of which begin with reading the problem aloud:

   Step 1 involves identifying what the problem is about (marbles; concert tickets; a rectangle).

   Step 2 asks students to make explicit what information they are supposed to find (a number of marbles; amounts of two kinds of tickets, each with a different price; the length and width of a rectangle in given units of measurement). Students answer these questions together, both verbally and in writing.

   Step 3 involves a scaffolded set of questions leading to the development of a diagram that represents both the known and unknown information about the quantities in the situation.

   Step 4 asks students to try to act out the problem using real objects to represent the quantities in the situation.

3. Pairs of students should present their diagrams to the class. As they view and interpret other pairs' diagrams, they add details or labels to their own diagrams.

Process outline

1. Students work individually on the problem.

2. Students form pairs and each pair shares one copy of the handout.

3. Pairs of students talk together to answer the questions in Steps 1–3 on the handout in writing.

4. Finally students try to act out the problem using physical objects to represent the quantities in the situation.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the information and the meaning of the problem themselves.

The structure of the task is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

Name _____

Reading and Understanding Creating Equations: Questions 2a, 2b, 2c

Name of the problem _____ (Question 2a, Question 2b, or Question 2c)

Step 1. Partner 1 reads the problem out loud to Partner 2. Answer the question together in writing.

> What's the problem about?

Step 2. Partner 2 reads the problem out loud to Partner 1. Answer the questions together in writing.

> What is the question in the problem? What are you looking for?

Step 3. Read the problem aloud a third time together. Talk to your partner and answer these questions together in writing.

> a. In mathematics problems, we often count or measure things. What are we counting or measuring in this problem?

b. What do you know about the quantities in this problem?

c. What unknown information do you need to find in order to solve the problem?

d. What operations and/or formulas are useful in this problem?

e. Draw a diagram of the problem and label all the information you know. Then try to represent all of the unknown information in your diagram.

Step 4. Use physical objects to represent the quantities in the situation (for example, number of marbles or tickets) and try to act the problem out using the objects.

MATHEMATICALLY SPEAKING: CREATING EQUATIONS

Adapted from R. Santa Cruz (2012) for the Understanding Language Project

Note

Students can first work on the Reading and Understanding Creating Equations task first to ensure that all are ready to begin working on each question in Creating Equations.

Purpose

The purpose of this Mathematically Speaking task is to provide students an opportunity to use important vocabulary orally and in writing *during* or *after* working on a mathematics task. It is intended as a *vocabulary review* and is not meant to be used to preview vocabulary. This task gives students the opportunity to first work on solving a mathematics problem, then to use targeted vocabulary to explain to a partner how they arrived at their solution; moreover, it gives students practice tracking and interpreting vocabulary used by their peers. It is crucial that students do this vocabulary work *after* they solve a mathematics problem that grounds the meanings for words. It is important to recognize that students will use everyday words in their talk while solving the mathematics problem and should not be corrected. Instead the teacher can provide more formal mathematical terms later during a whole class discussion.

Note that developing academic language is more than just learning the target or specialized vocabulary of a problem, lesson, unit, or chapter. For example, comparative structures such as "twice as many" or "3 less than" are syntactic structures that students need to understand and use not separated from solving a math problem, but at the same time as they are working on solving a math problem, so that use the target vocabulary for the purpose of communicating their reasoning.

Required for use

1. For each student: One or more mathematics tasks with solution strategies that require several sentences to describe and explain. (In this case, Creating Equations is used.)

2. For each pair of students: Mathematically Speaking tally charts with target vocabulary words. (In this case, target vocabulary is from Creating Equations.)

Structure of the activity

1. Students first work individually on the four questions in Creating Equations, with the requirement that they begin every problem, even if they are unable to get very far. They record their work on the handout, using additional space as needed, in preparation for working with a partner.

2. Student pairs are formed, and students work together to solve each problem, sharing their individual notes and each making new notes as they work.

3. Each pair then gets one copy of the tally sheet with target vocabulary words. One student explains his or her solution strategy for question 1 to the other student. The listener marks the tally sheet each time a target word is used in the explanation. If a target word is not used in the speaking student's initial explanation, the listening student can ask questions that use the target word(s) for further explanation. (For example, for question 1, target word *equivalent*: "What is equivalent to what in the given equation?" or "What is equivalent to what in your new equation?" For question 2b, target word s*olve*: "How did you know when you were ready to solve your equation?" or "How did you know you had finished solving your equation?") The listening students should encourage the speaking students to keep talking until all target words on the list have been used.

4. Students move on to the next question, switching roles as speaker and listener.

5. The pairs can add words to the tally sheet that come up in their explanations that they think are important or challenging, then share these with the class at the end of the activity.

Process outline

1. Each student receives a copy of the first handout and uses that to show their work on the problems.

2. Students form pairs to finish solving all problems together.

3. Each pair shares a copy of the tally sheet, tallying uses of target vocabulary words for each explanation, and adding words as needed.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the information and the meaning of the problem themselves.

The structure of the task is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

The teacher poses questions, takes notes, observes students, and asks students if they have questions or works with small groups with differentiated teaching.

Mathematically Speaking: Individual Work

Name: _____

Solve each question part. Show all your work and use more space on the back page or additional paper if needed.

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1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables

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2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

2b. Dave sold 40 tickets for a concert. He sold x tickets at \$2 each and y tickets at \$3 each. He collected \$88.

Write two equations connecting x and y.

2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2.

Write a quadratic equation, and solve it to find the length and width of this rectangle.

Mathematically Speaking: Pair Work

Partner Names: _____ & _____

Task Name: _____ Creating Equations _____

1. Partner 1 writes his or her name in the chart and explains solution strategy for Question 1.

2. Partner 2 listens to the explanation and marks a tally on the sheet each time Partner 1 uses a target word.

3. Partner 2 writes his or her name in the chart and explains solution strategy for Problem 2a while Partner 1 marks tallies on the sheet.

4. Switch roles again for Problem 2b, and again for Problem 2c.

5. Both partners add words to the chart that are important in the explanations.

Question 1. Target Words	Name _____	Question 2a. Target Words	Name _____
equation		equation	
variable		unknown	
equivalent		known	
expression		solve	
in terms of		solution	
		expression	
		more than	
		two times as many	

Question 2b. Target Words	Name _____	Question 2a. Target Words	Name _____
equation		equation	
unknown		unknown	
known		known	
solve		solve	
solution		solution	
expression		expression	
coefficient		length	
sum		width	
		factor	
		binomial	
		quadratic	

Understanding Language | Language, Literacy, and Learning in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet the Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS).

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